Pseudo Parity-Time Symmetry in Optical Systems

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We introduce a novel concept of the *pseudo* parity-time ($\mathcal{PT}$) symmetry in periodically modulated optical systems with balanced gain and loss. We demonstrate that whether the original system is $\mathcal{PT}$-symmetric or not, we can manipulate the property of the $\mathcal{PT}$ symmetry by applying a periodic modulation in such a way that the effective system derived by the high-frequency Floquet method is $\mathcal{PT}$ symmetric. If the original system is non-$\mathcal{PT}$ symmetric, the $\mathcal{PT}$ symmetry in the effective system will lead to quasi-stationary propagation that can be associated with the *pseudo* $\mathcal{PT}$ *symmetry*. Our results provide a promising approach for manipulating the $\mathcal{PT}$ symmetry of realistic systems.

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Parity-time ($\mathcal{PT}$) symmetry is an important concept in physics recently developed in application to optical systems. In quantum mechanics, the fundamental requirement of Hermitian Hamiltonians guarantees the existence of real eigenvalues and probability conservation. However, it was found recently that a wide class of non-Hermitian Hamiltonians with the (parity-time) $\mathcal{PT}$ symmetry can still possess entirely real eigenvalue spectra [1–4]. Although the extension of quantum mechanics based upon non-Hermitian but $\mathcal{PT}$-symmetric operators is still a subject to debates, optical systems with complex refractive indices [5–12] are widely used to test the $\mathcal{PT}$ symmetry in non-Hermitian systems, because of the equivalence between the Schrödinger equation in quantum mechanics and the wave equation in optics [13]. In the last few years, the $\mathcal{PT}$ symmetry was observed in several optical systems with gain and loss, such as optical couplers [14, 15], microwave billiard [16], and large-scale temporal lattices [17].

Similar to the electron transport in periodic crystalline potentials and the quantum tunneling in periodically driven systems [18], the light propagation in waveguides can be effectively controlled by periodic modulations [19–23]. In an optical system, periodic modulation is associated with a periodic refractive index. Mathematically, an optical system with periodic complex refractive index is equivalent to a time-periodic non-Hermitian quantum system. It has been theoretically demonstrated that high-frequency diagonal modulations can effectively rescale the natural coupling strength [18]. Therefore, a periodically modulated Hamiltonian can be mapped into a time-independent one. Like the unmodulated system, the $\mathcal{PT}$ symmetry may appear in the effective system if the periodically modulated system may be described by a $\mathcal{PT}$-symmetric Hamiltonian [24]. Naturally, an important question arises: *Can the $\mathcal{PT}$ symmetry appears in an effective system even if the periodically modulated system is non-$\mathcal{PT}$-symmetric?* In other words, can we employ periodic modulations to manipulate the $\mathcal{PT}$ symmetry?

In this Letter, we study the light propagation in a periodically modulated optical coupler with balanced gain and loss and apply a bi-harmonic modulation along the propagation direction. The Hamiltonian for the modulated system is non-$\mathcal{PT}$-symmetric if the relative phase between the two applied harmonics is not 0 or π. Applying the high-frequency Floquet approach, the modulated system is effectively described by an effective averaged system, whose $\mathcal{PT}$ symmetry can be manipulated by tuning the modulation amplitude/frequency. More importantly, the $\mathcal{PT}$ symmetry can appear in the effective system corresponding to a non-$\mathcal{PT}$-symmetric and non-Hermitian Hamiltonian. Different from the $\mathcal{PT}$ symmetry from a $\mathcal{PT}$-symmetric Hamiltonian, which leads to stationary light propagation of bounded intensity oscillation, the $\mathcal{PT}$ symmetry from a non-$\mathcal{PT}$-symmetric Hamiltonian will lead to quasi-stationary light propagation of unbounded intensity oscillation. Therefore, we term the induced symmetry associated with modulated systems as the *pseudo* $\mathcal{PT}$ *symmetry*.

In optics, the electric field $E(x,z)$ of light obeys the wave equation,

$$i\frac{\partial E(x,z)}{\partial z} = -\frac{1}{2k}\frac{\partial^2 E(x,z)}{\partial x^2} + V(x,z)E(x,z), \qquad (1)$$

where, $k = k_0 n_0$, $k_0 = 2\pi/\lambda$ and $V(x,z) = k_0[n_0 - n(x)]$ with the substrate index n_0, the free-space wavelength

λ and the complex refractive index distribution $n(x) = n_0 + n_R(x,z) + in_I(x)$, where n_R and n_I are real and imaginary parts of the refractive index. Therefore, the effective potential reads as $V(x,z) = V_R(x,z) + iV_I(x) = -k_0[n_R(x,z) + in_I(x)]$. With the experimental techniques developed in recent years [6, 10, 12, 14, 15], one can make $V_I(-x) = -V_I(x)$ and $V_R(x,z) = V_0(x) + V_1(x,z)$ with the unmodulated part $V_0(x)$ being a symmetric double-well function and the modulation $V_1(x,z) = V'(x)F(z)$ described by an anti-symmetric function $V'(-x) = -V'(x)$ and a bi-harmonic function $F(z)$, see Fig. 1. Using the coupled-mode theory, the electric field for a two-channel coupler can be expressed as $E(x,z) = c_1(z)\psi_1(x) + c_2(z)\psi_2(x)$ with the two transverse distributions $\{\psi_1(x), \psi_2(x)\}$ and the two field amplitudes $\{c_1(z), c_2(z)\}$. Integrating Eq. (1) with respect to x, we find $\{c_1(z), c_2(z)\}$ obeying

$$i\frac{d}{dz}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} +\frac{i\gamma}{2} + \frac{S(z)}{2} & v \\ v & -\frac{i\gamma}{2} - \frac{S(z)}{2} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}, \quad (2)$$

with the dimensionless propagation distance z, the inter-channel coupling strength v, the gain/loss strength γ and the bi-harmonic modulation

$$S(z) = -A[\sin(\omega z) + f\sin(2\omega z + \phi)]. \quad (3)$$

Here, $\phi \in [0, 2\pi)$ denotes the relative phase between the two harmonics, ω is the modulation frequency, A is the modulation amplitude and f is a dimensionless coefficient. Since the system is invariant under the transformation $c_2 \to -c_2$ and $v \to -v$, below we will only consider the case of $v > 0$. Defining the parity operator as $\hat{P}$, which interchanges the two channels labeled by 1 and 2, and the time operator as $\hat{T}$: $i \to -i, z \to -z$, which reverses the propagation direction, the Hamiltonian $\hat{H}(z)$ for the modulated system (2) is $\mathcal{PT}$ symmetric if $\hat{P}\hat{T}\hat{H} = \hat{H}\hat{P}\hat{T}$. If $\phi = 0$ or π, $S(-z) = -S(z)$, $\hat{H}(z)$ is $\mathcal{PT}$-symmetric. Otherwise, if $\phi \neq 0$ and π, $S(z_0 - z) \neq -S(z_0 + z)$ for any arbitrary constant z_0, $\hat{H}(z)$ becomes non-$\mathcal{PT}$-symmetric.

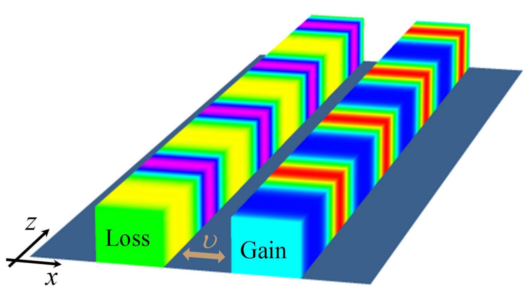

FIG. 1: Schematic diagram of a periodically modulated two-channel optical coupler with balanced gain and loss. The periodic change of color along z-axis denotes the periodic modulation.

Under the condition of $\omega \gg \{v, \gamma\}$, one can implement the high-frequency Floquet treatment. Introducing the transformation

$$c_1 = c_1' \exp\left\{ -i\left[\frac{A}{2\omega}\cos(\omega z) + \frac{Af}{4\omega}\cos(2\omega z + \phi)\right]\right\}, \quad (4)$$

$$c_2 = c_2' \exp\left\{ +i\left[\frac{A}{2\omega}\cos(\omega z) + \frac{Af}{4\omega}\cos(2\omega z + \phi)\right]\right\}, \quad (5)$$

and averaging the high frequency terms, one can obtain an effectively unmodulated system

$$i\frac{d}{dz}\begin{pmatrix} c_1' \\ c_2' \end{pmatrix} = \begin{pmatrix} +\frac{i\gamma}{2} & J \\ J^* & -\frac{i\gamma}{2} \end{pmatrix}\begin{pmatrix} c_1' \\ c_2' \end{pmatrix}, \quad (6)$$

with the effective coupling strength

$$J = v\sum_{m=-\infty}^{\infty} (i)^{-m} J_{-2m}\left(\frac{A}{\omega}\right) J_m\left(\frac{Af}{2\omega}\right)\exp(im\phi). \quad (7)$$

The modulus of J depends on the values of A/ω and ϕ. If A/ω is relatively small, the modulus $|J|$ is almost independent on the relative phase ϕ. When A/ω increases, the modulus $|J|$ becomes sensitively dependent on ϕ. In particular, the modulus $|J|$ equals to zero at some specific values of A/ω (such as $A/\omega \simeq 2.4$ and 5.52) and $\phi = \pi/2$ or $3\pi/2$. In Fig. 2 (a), choosing $f = 1/4$, we show the contour plot of $|J|$ as a function of A/ω and ϕ.

By diagonalizing the Hamiltonian for the effective system (6), the two eigenvalues are given as

$$\varepsilon = \pm|J|\left[1 - \left(\frac{\gamma}{2|J|}\right)^2\right]^{1/2}. \quad (8)$$

Obviously, dependent on the values of $\frac{\gamma}{2|J|}$, the two eigenvalues can be real or complex. The two eigenvalues are real if $\gamma < 2|J|$ and they become complex if $\gamma > 2|J|$. Therefore, $\gamma_{critical} = 2|J|$ is the critical point for the phase transition between real and complex spectra in the effective system, which corresponds to the original system (2) under high-frequency modulations. The spontaneous $\mathcal{PT}$-symmetry-breaking transition takes place in the effective model (6) when the imaginary part of ε changes from zero to nonzero. Surprisingly, unlike our conventional understanding, we find that the quasi-energies can be real even if the modulated system (2) is non-$\mathcal{PT}$-symmetric (i.e. $\phi \neq 0$ and π).

The parametric dependence of $|\mathrm{Im}(\varepsilon)|$ is shown in Fig. 2 (b-e). In Fig. 2 (b-c), we show $|\mathrm{Im}(\varepsilon)|$ as a function of γ and ϕ for $f = 1/4$. For small A/ω, such as $A/\omega = 1$ in Fig. 2 (b), $|\mathrm{Im}(\varepsilon)|$ is almost independent on the phase ϕ and the transition from a completely real quasi-energy spectrum ($|\mathrm{Im}(\varepsilon)| = 0$) to a complex spectrum ($|\mathrm{Im}(\varepsilon)| \neq 0$) take places when γ increases. Near a minimum of $|J|$, such as $A/\omega = 2.4$, $|\mathrm{Im}(\varepsilon)|$ strongly depends on the phase ϕ, see Fig. 2 (b). In Fig. 2 (d-e), we show $|\mathrm{Im}(\varepsilon)|$ as a function of γ and A/ω for (d) $\phi = 0$ and (e) $\phi = \pi/2$. Near the minima of $|J|$, such as $A/\omega \simeq 2.4, 5.52, ...$, $|\mathrm{Im}(\varepsilon)|$ shows significant difference between

the two cases of $\phi = 0$ and $\phi = \pi/2$. In particular, at the minimum points, $|J|$ vanished to zero for $\phi = \pi/2$ and the corresponding critical value $\gamma_{critical} = 2|J|$ is reduced to zero. Similar to a non-Hermitian system with no modulations, the spontaneous $\mathcal{PT}$-symmetry-breaking transition ($|\mathrm{Im}(\varepsilon)| = 0 \Rightarrow |\mathrm{Im}(\varepsilon)| \neq 0$) can be observed by tuning the gain/loss strength γ. More interestingly, for our modulated system (2) of fixed γ, it is possible to observe the spontaneous $\mathcal{PT}$-symmetry-breaking transition by tuning the modulation parameters ϕ and A/ω, see Fig. 2 (c-e).

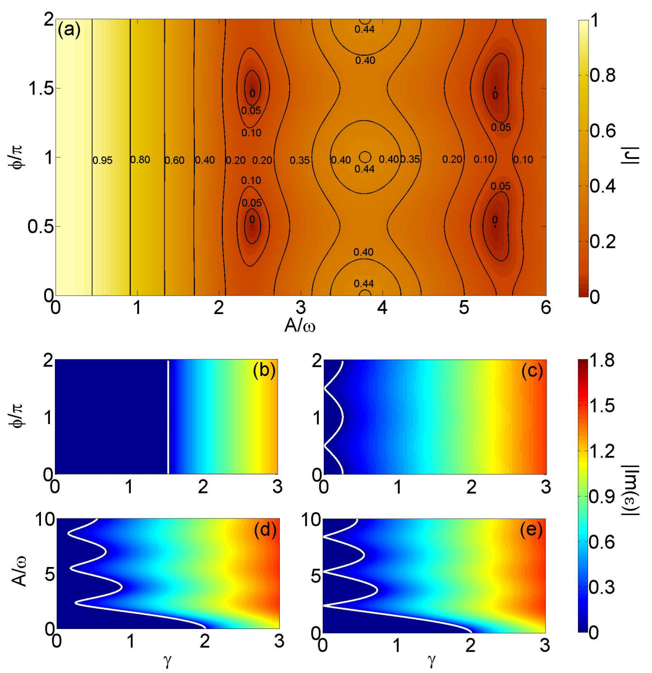

FIG. 2: (color online) The parametric dependence of the effective coupling $|J|$ and the imaginary parts of quasi-energies $|\mathrm{Im}(\varepsilon)|$. Top row [(a)]: $|J|$ versus A/ω and ϕ for $f = 1/4$. Middle row [(b) and (c)]: $|\mathrm{Im}(\varepsilon)|$ versus ϕ/π and γ for (b) $A/\omega = 1$ and (c) $A/\omega = 2.4$. Bottom row [(d) and (e)]: $|\mathrm{Im}(\varepsilon)|$ versus A/ω and γ for (d) $\phi = 0$ and (e) $\phi = \pi$. The other parameters for $|\mathrm{Im}(\varepsilon)|$ are chosen as $v = 1$, $\omega = 10$ and $f = 1/4$. The white curves are the boundary ($\gamma_{critical} = 2|J|$) between $|\mathrm{Im}(\varepsilon)| = 0$ and $|\mathrm{Im}(\varepsilon)| \neq 0$.

Based upon the high-frequency Floquet treatment, it seems that, whether the modulated system (2) obeys a $\mathcal{PT}$-symmetric Hamiltonian or not, completely real quasi-energy spectrum always appear if $\gamma < 2|J|$. This is obviously inconsistent with the previous theory [1–4] which tells us that only $\mathcal{PT}$-symmetric Hamiltonian systems can support completely real spectra. So, what really happens in the modulated non-Hermitian and non-$\mathcal{PT}$-symmetric Hamiltonian system?

In general, for systems under temporally periodic modulations of arbitrary modulation frequencies, one can use the Floquet theory to calculate their quasi-energies. Similar to the Bloch states for systems of spatially periodic potentials, the modulated system (2) has Floquet states, $\{c_1(z), c_2(z)\} = e^{-i\varepsilon z}\{\tilde{c}_1(z), \tilde{c}_2(z)\}$. Here, the propagation constant ε is called as the quasi-energy, and the two complex amplitudes $\tilde{c}_1(z)$ and $\tilde{c}_2(z)$ are periodic with the modulation period $T = 2\pi/\omega$.

To check the validity of the effective model (6), we calculate numerically the quasi-energies for the original model (2) and then compare the numerical results with the analytical formula (8). Under the condition of $\omega = 10$, $v = 1$ and $f = 1/4$, the analytical formula (8) is well consistent with the numerical results for γ up to 3. The analytical and numerical values for the quasi-energies ε are in good agreement and only show tiny difference dependent upon the phase ϕ. As two examples, we show $\mathrm{Im}(\varepsilon)$ (the imaginary part of quasi-energy) versus γ for $\phi = 0$ and $\phi = \pi/2$ in Fig. 3 (a) and (b), respectively. It clearly shows that the analytical results (red lines) agree well with the numerical results (black lines) obtained from the original model (2). Below the critical point $\gamma_{critical} = 2|J|$, for systems described by $\mathcal{PT}$-symmetric Hamiltonians (i.e. $\phi = 0$ or π), the numerical results confirm the entirely real quasi-energy spectrum, see Fig. 3 (c). However, for systems described by non-$\mathcal{PT}$-symmetric Hamiltonians (i.e. $\phi \neq 0$ and π), the numerical results show the quasi-energies ε still have small non-zero imaginary parts even below the critical point $\gamma_{critical} = 2|J|$, see Fig. 3 (d). This means that, if the original system (2) obeys a non-$\mathcal{PT}$-symmetric Hamiltonian, the entirely real quasi-energy spectrum for the effective model (6) does not correspond to perfectly entirely real quasi-energy spectrum for the original system (2). Therefore, such a $\mathcal{PT}$ symmetry in the effective model (6) corresponds to a kind of *pseudo $\mathcal{PT}$ symmetry* in the original model (2).

Through numerical integration, we analyze the light propagations in the modulated system (2) of different quasi-energies corresponding to different symmetries. The light propagation sensitively depends upon the quasi-energies. Stationary light propagations of bounded intensity oscillations appear if all quasi-energies are real. Non-stationary light propagations of unbounded intensity oscillations appear if at least one of quasi-energies is complex, in which quasi-stationary light propagations of slowly varying time-averaged intensities appear if the two quasi-energies for the effective system (6) are real. In Fig. 4, for $v = 1$, $A = 10$, $f = 1/4$, $\omega = 10$ and $\gamma = 0.1$ (which is below the critical value $\gamma_{critical}$), we show the intensity evolution from the initial state of $c_1(0) = 1$ and $c_2(0) = 0$. In which, the two intensities $I_j(z) = |c_j(z)|^2$, the total intensity $I_t(z) = I_1(z) + I_2(z)$ and the time-averaged total intensity $I_t^{av}(z) = \frac{1}{T_s}\int_z^{z+T_s} I_t(\tilde{z})d\tilde{z}$ with $T_s = 2\pi/|\mathrm{Re}(\varepsilon_2) - \mathrm{Re}(\varepsilon_1)|$ and $\mathrm{Re}(\varepsilon_j)$ being the real part of ε_j. In short-distance propagations, $I_{1,2}(z)$ and $I_t(z)$ oscillate periodically and it is hard to see the difference between the cases of $\phi = 0$ and $\pi/2$, see Fig.

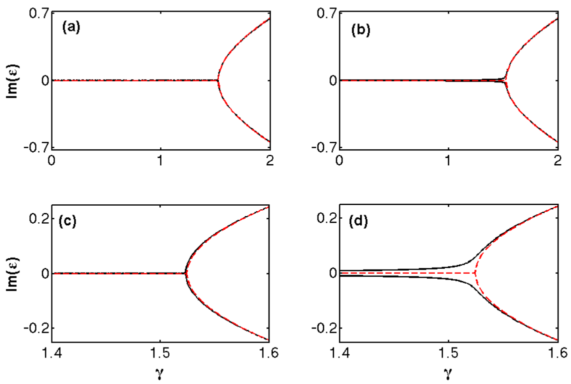

FIG. 3: (color online) Comparison between numerical and analytical results of $\mathrm{Im}(\varepsilon)$, the imaginary part of quasi-energy. Upper row: $\mathrm{Im}(\varepsilon)$ versus γ for (a) $\phi = 0$ and (b) $\phi = \pi/2$. Solid lines are for numerical results obtained from the original model (2) and red dashed lines are analytical results given by the formula (8) for the effective model (6). Lower row: the enlarged regions of (a) and (b) near the bifurcation point given by the analytical formula (8). The other parameters are $v = 1$, $A = 10$, $f = 1/4$ and $\omega = 10$.

4 (c-d). However, significant difference appears in long-distance propagations. For a $\mathcal{PT}$-symmetric Hamiltonian system (2) of $\phi = 0$, the time-averaged total intensity $I_t^{av}(z)$ keeps unchanged, see Fig. 4 (a). For a non-$\mathcal{PT}$-symmetric Hamiltonian system (2) of $\phi = \pi/2$, the time-averaged total intensity $I_t^{av}(z)$ slowly increases, see Fig. 4 (b). The quasi-stationary light propagations of slowly varying $I_t^{av}(z)$ is a direct signature of the *pseudo* $\mathcal{PT}$ *symmetry* in the modulated system (2).

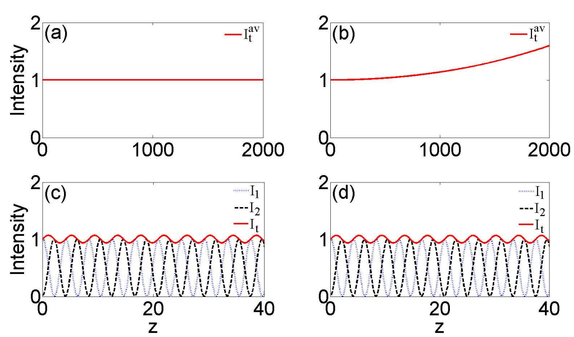

FIG. 4: (color online) Intensity evolution from the initial state of $c_1(0) = 1$ and $c_2(0) = 0$. Upper row : long-distance time-averaged intensity evolution for (a) $\phi = 0$ and (b) $\phi = \pi/2$. Lower row: short-distance intensity evolution for (c) $\phi = 0$ and (d) $\phi = \pi/2$. The other parameters are chosen as $v = 1$, $A = 10$, $f = 1/4$, $\omega = 10$ and $\gamma = 0.1$.

Now, we discuss the experimental possibility to observe our theoretical predictions. Recently, several $\mathcal{PT}$-symmetric optical systems have been realized experimentally [6, 10, 12, 14, 15]. Complex refractive index of gain/loss effects can be obtained from quantum-well lasers or photorefractive structures through two-wave mixing [25]. Periodic modulations can be introduced by out-of-phase harmonic modulations of the real refractive index [13, 21, 23] or periodic curvatures along the propagation direction [13, 19, 23, 26]. For a short optical coupler under periodic modulations, spontaneous $\mathcal{PT}$-symmetry-breaking transitions can be observed, whether the system Hamiltonian is $\mathcal{PT}$-symmetric or not. In such a system, the light propagation will be periodic and stable if $\gamma < 2|J|$, and an instability will be observed if $\gamma > 2|J|$. The critical point $\gamma_{critical} = 2|J|$ can be adjusted by controlling the modulation parameters A/ω, f and ϕ in addition to controlling the gain/loss strength γ. However, for a long optical coupler under periodic modulations, the light propagation below the critical point (i.e. $\gamma < 2|J|$) depends on the Hamiltonian symmetry. If the Hamiltonian is $\mathcal{PT}$-symmetric, the light propagation is periodic and stable, in which the time-averaged total intensity keeps conserved. Otherwise, if the Hamiltonian is non-$\mathcal{PT}$-symmetric, the light propagation is quasi-stationary, in which the time-averaged total intensity slowly changes.

In summary, we have studied the non-Hermitian Hamiltonian systems under periodic modulations and introduce the concept of the *pseudo* $\mathcal{PT}$ *symmetry*. If the modulated system obeys a $\mathcal{PT}$ symmetric Hamiltonian, there exists a truly spontaneous $\mathcal{PT}$-symmetry-breaking phase transition from a real quasi-energy spectrum to a complex one. If the modulated system obeys a non-$\mathcal{PT}$-symmetric Hamiltonian, although there exists a spontaneous $\mathcal{PT}$-symmetry-breaking phase transition in the effective system derived from the high-frequency Floquet treatment, there is no truly spontaneous $\mathcal{PT}$-symmetry-breaking phase transition in the original system. Corresponding to the real spectrum for the effective system, the original system has a quasi-real spectrum of small imaginary parts, which leads to a quasi-stationary light propagation of slowly varying time-averaged total intensity. This is the *pseudo* $\mathcal{PT}$ *symmetry* in the non-Hermitian system described by a non-$\mathcal{PT}$-symmetric Hamiltonian.

In addition to the discovery of the *pseudo* $\mathcal{PT}$ *symmetry*, we believe that our work brings three key advances to related fields. Firstly, although the high-frequency Floquet treatment can capture most key features, some important information (such as the so-called *pseudo* $\mathcal{PT}$ *symmetry*) may be lost. Secondly, periodic modulations provide a new route to the observation of the spontaneous $\mathcal{PT}$-symmetry-breaking transition. Thirdly, as the inter-channel coupling can be effectively switched off by controlling the modulation and the corresponding intensity grows exponentially even for arbitrarily weak gain/loss, this exponential growth offers an efficient way to beam amplification in optical waveguides.

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